Table 2 shows a similar comparison for blowing with $f(0) = 0$ to -0.278.

4. DISCUSSION

Equation (15) can be shown to be the first term in the series obtained if an exact solution to equation (7) is sought using singular perturbation methods (for example, as was done by Walker *et al.* [9]). It is accurate for $Sc_p \gtrsim 10^3$, and thus can be regarded as an exact solution for the present purpose. Equation (20) is an approximate result obtained by simply adding the deposition rates for each mechanism if they are assumed to act independently, and also if it is assumed that thermophoretic deposition can be calculated using the temperature gradient of a stagnant film model, i.e. $\partial T/\partial y$ in equation (2) is approximated as $(T_s - T_e)/\delta_h$, $\delta_h = k/h_c$, to obtain equation (17b). Tables 1 and 2 show that the agreement is good only when convection dominates, e.g. $f(0) = 5.0$, $\kappa = 0.01$ in Table 1. However, the tables also show that when thermophoresis dominates, e.g. $f(0) = 0$, $\kappa = 1.0$ in Table 1, the agreement is poor. This discrepancy is due to the failure of the approximate method to recognize that Brownian diffusion does play an important role in the inner region of the concentration boundary layer, even for large Sc_p and κ: since $N \to 0$ at the wall, Brownian diffusion dominates as $y \to 0$. The approximate method in using the stagnant film model to give an average temperature gradient for the calculation of the thermophoretic velocity [equation (17b)], overestimates the rate of deposition by nearly 50% for $\kappa = 1.0$.

For the condensation of superheated steam–noncondensable gas mixtures, both κ and $f(0)$ can be large, and Table 1 shows that the approximate equation can overestimate deposition rates by almost 100%. A situation of particular importance to boiling water nuclear reactor safety is the scrubbing of aerosol particles from nearly pure gas bubbles rising through a saturated suppression pool: in such a case there is evaporation into the bubbles and the only removal mechanism is thermophoresis: since the removal rates are then very low the situation is particularly dangerous. Table 2 shows that for low evaporation rates, $0 < -f(0) < 0.01$, the approximate equation overestimates the removal rate by nearly 50%.

Equation (19a) is exact and allows an unambiguous evaluation of the coupling between thermophoresis and convection. However, choice of an appropriate 'blowing factor' for the particle Stanton number, and hence the equivalent stagnant film thickness for Brownian diffusion, is not straightforward. Possibilities which have, or may be considered, for use in industry include (1) unity, e.g. refs. [1, 3], (2) the factor appropriate for the limit $Sc_p \to \infty$, as given by Bird *et al.* [10], and (3) the same factor as that used for heat transfer, i.e. equation (19b), as used herein. Our choice was somewhat arbitrary, but the important conclusions of our study are not altered if an alternative choice is made.

In conclusion we suggest that the results displayed here indicate that current engineering methods of calculating deposition rates of small aerosol particles can be significantly in error, and that proper accounting for the coupling between the various deposition mechanisms is required to obtain improved results.

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HEAT TRANSFER BETWEEN HOT COMBUSTION GASES AND A COLD WALL IN NARROW CHANNELS FOR LIMIT FLAMES

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NOMENCLATURE

c_p	specific heat at constant pressure
d	the 4/5 length of the high temperature zone
D	width of the narrow channel, equal to the quenching distance for limit flames
Nu	Nusselt number as related to the quenching distance D, $\alpha D/\lambda_b$
Pe	Péclet number expressed in terms of the parameters of the hot combustion gases for limit flames, $c_{pb}\rho_b u_b D/\lambda_b$
Pe_L	Péclet number expressed in terms of the parameters of the fresh mixture for limit flames, $c_{pu}\rho_u u_L D/\lambda_u$
St	Stanton number, Nu/Pe
T	absolute temperature
u	flow speed
u_L	burning velocity
x	dimensionless coordinate normal to the flame front
z	distance normal to the flame front.

Greek symbols
- α heat transfer coefficient
- β exponent in equation (7) defined by relation (8)
- λ thermal conductivity
- ρ density
- τ dimensionless temperature
- ϕ equivalence ratio
- ψ dimensionless coefficient, $2Nu/Pe^2$.

Subscripts
- b burned
- u unburned.

THE HEAT transfer between hot combustion gases and a cold wall, in the course of a process of free propagation of a flame in a tube, is usually estimated on the basis of an approximate empirical relation [1]. In ref. [2] the length of the hot region behind the flame front of methane–air mixtures in a narrow channel was measured for limit flames. The results will be used here to determine the real values of the heat transfer parameters.

The considerations are confined to the case of a flame propagating downwards in a narrow channel between two walls. The aim of the measurements was to determine the structure of the flame in narrow channels under conditions approaching those of quenching. The quenching conditions are expressed by the equivalence ratios (or the laminar burning velocities) and the associated distances between the walls, critical for the existence of the flame. These conditions are characterized by a constant Péclet number $Pe_L = 39$ [2]. The term 'limit flame' will be used to denote a flame maintained close to the quenching conditions.

The temperature profiles which had been measured enabled the differences between the flame structure in a narrow and a wide channel to be revealed (Fig. 1). The nature of those differences is that the combustion gases behind the flame in a narrow channel are almost immediately cooled, while in the case of a wide channel a core of hot combustion gases at a temperature approaching the maximum temperature is maintained over a certain distance behind the flame front.

The existence of turbulence may be excluded in narrow channels (where it will be shown that $Re = 23$). The same applies to natural convection ($Re = 550$). This statement is confirmed by the temperature profiles of the combustion gases, which were found to be slender and continuous.

Limit flames in narrow channels are flat. In this connection the flow over the first segment behind the flame front may be treated in the same way as the development of a uniform flow downstream of the entry into a plane channel. The length of the hot region being small, it may be assumed that a velocity profile typical of laminar flow will not have enough time to be formed, especially because the cooling of the gas favours flatter velocity profiles than in the case of isothermal flow [3].

The temperature distributions which were obtained by measurement in cross-sections of narrow channels (an example of such measurements being given in ref. [2]) showed that the temperature in the core is almost uniform.

These considerations show that the application of a one-dimensional 'plug flow' energy equation to describe the temperature is an admissible approximation and, after making the additional assumptions that radiation can be neglected and that the wall temperature is T_u, it can be written as

$$c_{pb}\rho_b u_b \frac{dT}{dz} = \lambda_b \frac{d^2T}{dz^2} - \frac{2\alpha(T-T_u)}{D}, \tag{1}$$

where c_{pb} and λ_b are assumed to be constant.

The variables can be reduced to the nondimensional form

$$dx = \frac{c_{pb}\rho_b u_b}{\lambda_b} dz, \tag{2}$$

$$\tau = \frac{T-T_u}{T_b-T_u}. \tag{3}$$

On substituting equations (2) and (3), equation (1) can be expressed in the dimensionless form

$$\frac{d^2\tau}{dx^2} - \frac{d\tau}{dx} - \psi\tau = 0, \tag{4}$$

where

$$\psi = \frac{2Nu}{Pe^2}. \tag{5}$$

The boundary conditions are

$$x = 0, \quad \tau = 1, \qquad x = \infty, \quad \tau = 0. \tag{6}$$

A solution of equation (4) can be found in the form

$$\tau = e^{\beta x}, \tag{7}$$

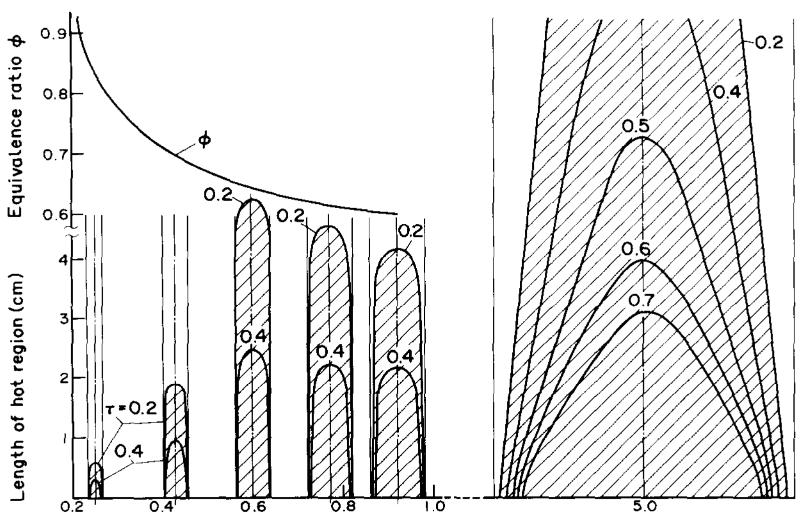

FIG. 1. The length of the hot region and the equivalence ratio as a function of the distance between the walls for limit flames.

where

$$\beta = \tfrac{1}{2}[1-\sqrt{(1+4\psi)}]. \tag{8}$$

If some temperature records are available; one can determine the exponent β and therefore also the relation between the parameters of flow and those of heat transfer

$$Nu = 2\beta^2\,Pe^2 - 2\beta\,Pe^2. \tag{9}$$

Since

$$2\beta^2\,Pe^2 \ll Nu, \tag{10}$$

one can write, finally

$$Nu \cong -2\beta\,Pe^2. \tag{11}$$

The temperature records of ref. [2] are similar up to 80% of their time scale, so the heat transfer up to the point at which the temperature becomes one fifth of its former value can be considered. According to equation (7) one obtains

$$e^{\beta x_1} = 0.2, \tag{12}$$

and

$$\beta x_1 = -1.62.$$

According to equation (2) one obtains

$$x_1 = \frac{Pe}{D}\,d,$$

and

$$\beta = -\frac{1.62}{2(d/D)\,Pe}. \tag{13}$$

On substituting this into equation (11) one finds

$$Nu = \frac{1.62}{d/D}\,Pe. \tag{14}$$

Relation (14) can also be expressed by introducing the Stanton number

$$\frac{Nu}{Pe} = \frac{1.62}{d/D} = St. \tag{15}$$

The Péclet number for the hot region of combustion gases in a narrow channel $Pe = c_{pb}\rho_b u_b D/\lambda_b$ can be easily determined, under quenching conditions, in terms of the Péclet number of the flame, which is constant under such conditions at $Pe_L = 39$ [2]

$$Pe = Pe_L\frac{c_{pb}}{c_{pu}}\frac{\lambda_u}{\lambda_b}, \tag{16}$$

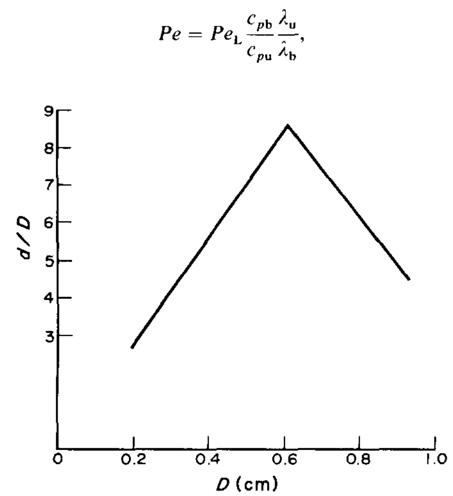

FIG. 2. Dimensionless length of the hot region near the quenching limit in a narrow channel, where d is the 4/5 length of the high temperature zone and D is the quenching distance.

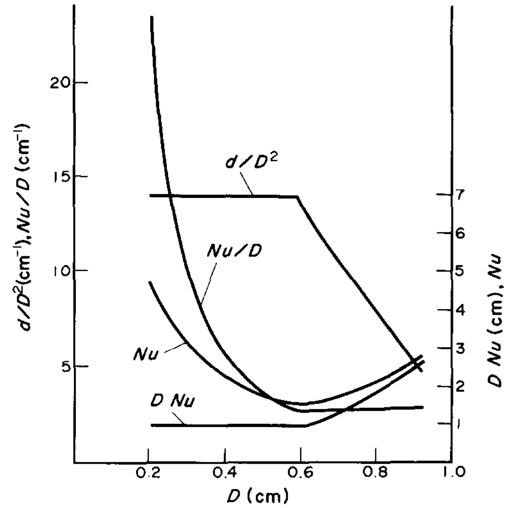

FIG. 3. Nusselt number as a function of the quenching distance D under limit conditions.

since according to the equation of conservation of mass

$$\rho_b u_b = \rho_u u_L. \tag{17}$$

The values of c_{pb} and λ_b used for the analysis should be the mean values over the temperature range $T_b - T_u$. Depending on the mixture composition in quenching channels, the differences in these temperatures vary from about $T_b - T_u = 1000°C$ for an extremely lean mixture to $T_b - T_u = 1700°C$ for a stoichiometric mixture, so the mean values of c_{pb} and λ_b will be somewhat different for different compositions. Taking a mean composition, one finds $(c_{pb}/c_{pu})(\lambda_u/\lambda_b) = 0.41$ and, finally, $Pe = 16$. It follows that the Reynolds number is also constant under the conditions considered, because $Re = Pe/Pr = 16/0.71 \cong 23$.

Since, under the limit conditions, the Péclet number was found to be constant, the parameter d/D expresses completely the variability of the Nusselt and the Stanton numbers. The experimental curve of ref. [2] can be replaced by the simplified curve expressed in Fig. 2. The variation of the Nusselt number Nu as a function of the quenching distance D is shown in Fig. 3.

It is seen that the experimental data lead to correlations which indicate that $D\,Nu \cong$ const. for $D \leqslant 0.6$ cm and

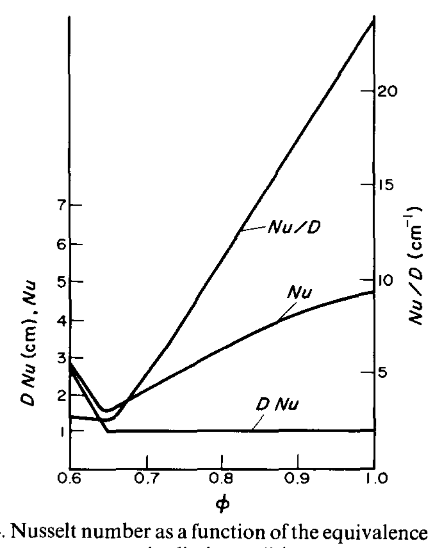

FIG. 4. Nusselt number as a function of the equivalence ratio ϕ under limit conditions.

$Nu/D \cong$ const. for $D \geqslant 0.6$ cm. The same parameters are shown in Fig. 4 as a function of the equivalence ratio under quenching conditions.

The case of $D\,Nu =$ const. is equivalent to that of λ/α $=$ const. $\cdot D^2$, and the case of $Nu/D =$ const. is equivalent to that of $\lambda/\alpha =$ const.

It is probable that those relations express the existence of a different behaviour of the reacting mixture under quenching conditions, rather than different heat transfer rules.

Thus, for instance, those relations can be explained by stating that the transition to $Nu/D =$ const. for $D \geqslant 0.6$ cm ($\phi < 0.65$) is accompanied by stabilization of the quantity of heat release, due to an increase in thickness of the wall layers in which the reaction is much slower or does not occur at all.

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MELTING RATES IN TURBULENT RECIRCULATING FLOW SYSTEMS

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NOMENCLATURE

C_p	specific heat of water
d	diameter of the ice rod
h	heat transfer coefficient
H	height of the tank
k	thermal conductivity of water
L	latent heat of melting
r	radial coordinate
r_c	radii of the jet cone at a given level
R	radius of the ice rod
dR/dt	melting rate of the ice rod
Re	Reynolds number, $U_{loc}d\rho/\mu$
T	temperature
Tu	turbulence intensity, defined in the text
U_0	centerline velocity
U'	fluctuating velocity
$\bar{U}$	time smoothed velocity
U_{loc}	local velocity
z	axial coordinate.

Greek symbols

μ	viscosity of water
ρ	density of ice.

1. INTRODUCTION

The purpose of this technical note is to report on experimental measurements concerning the rate at which ice rods melt, when immersed in a pool of water, agitated by an ascending stream of gas bubbles. Problems of this type are of both fundamental and practical interest. From a practical standpoint there are numerous metal processing operations where heat (or mass) transfer between an agitated melt and a solid surface play a central role in determining the overall feasibility or efficiency of the system. The rate at which ferrous or aluminum scrap melts in furnaces, the erosion (corrosion) of refractories in steel processing and the smelting of nickel ores may be cited as examples [1–3].

These problems are of interest from a fundamental standpoint, because while turbulent recirculating flows are reasonably well modelled at present, serious questions remain regarding the appropriate representation of the conditions in the proximity of solid boundaries [4–6].

In a recent paper the present authors described experimental measurements dealing with the velocity profiles and the maps of the turbulent kinetic energy in water, held in a cylindrical container, which was agitated by a gas stream, injected at an axi-symmetrical location at the bottom [7].

The experimental measurements were compared with theoretical predictions based on the numerical solution of the turbulent Navier–Stokes equations, in conjunction with the k model of the turbulent viscosity. In general there was good agreement between the measurements and the predictions.

The investigation to be described here is an extension of this previously described work.

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2. EXPERIMENTAL WORK

The apparatus consisted of a cylindrical tank, containing water which was agitated by a gas stream, introduced through the bottom of the container, via a centrally located orifice. Ice rods (frozen onto a steel supporting rod) were immersed into this agitated body of water and the rate of melting was established by photographic means. This cylindrical tank was surrounded by a square tank, also filled with water, in order to eliminate the parallax effects in the optical measurements.

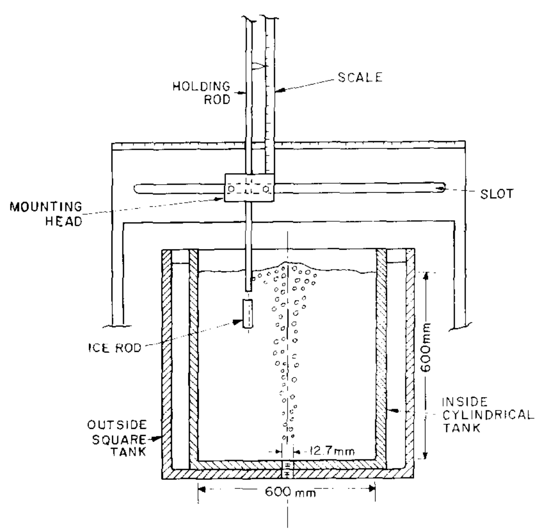

Fig. 1. Schematic sketch of the apparatus.